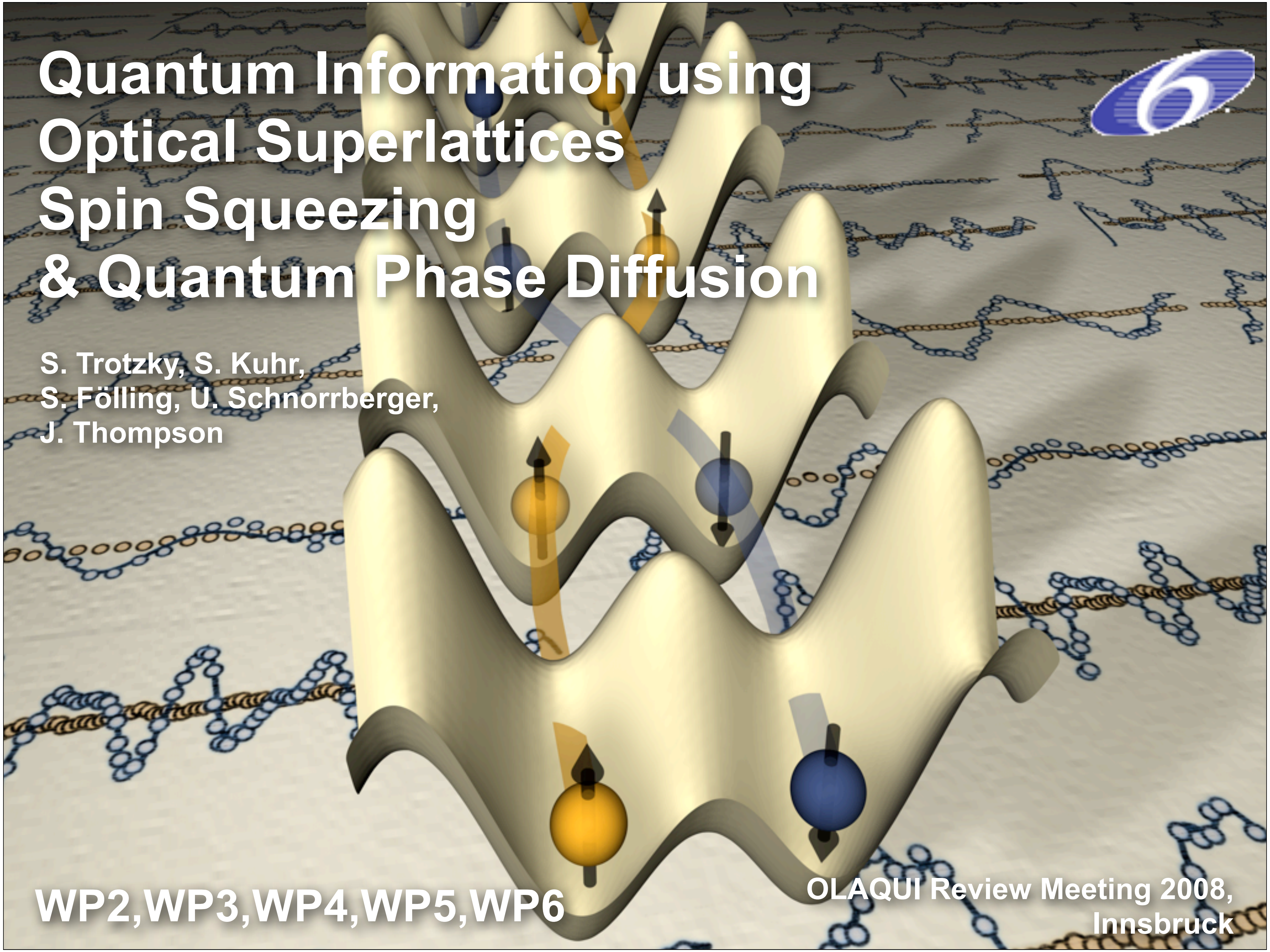


Quantum Information using Optical Superlattices
Spin Squeezing
& Quantum Phase Diffusion

S. Trotzky, S. Kuhr,
S. Fölling, U. Schnorrberger,
J. Thompson

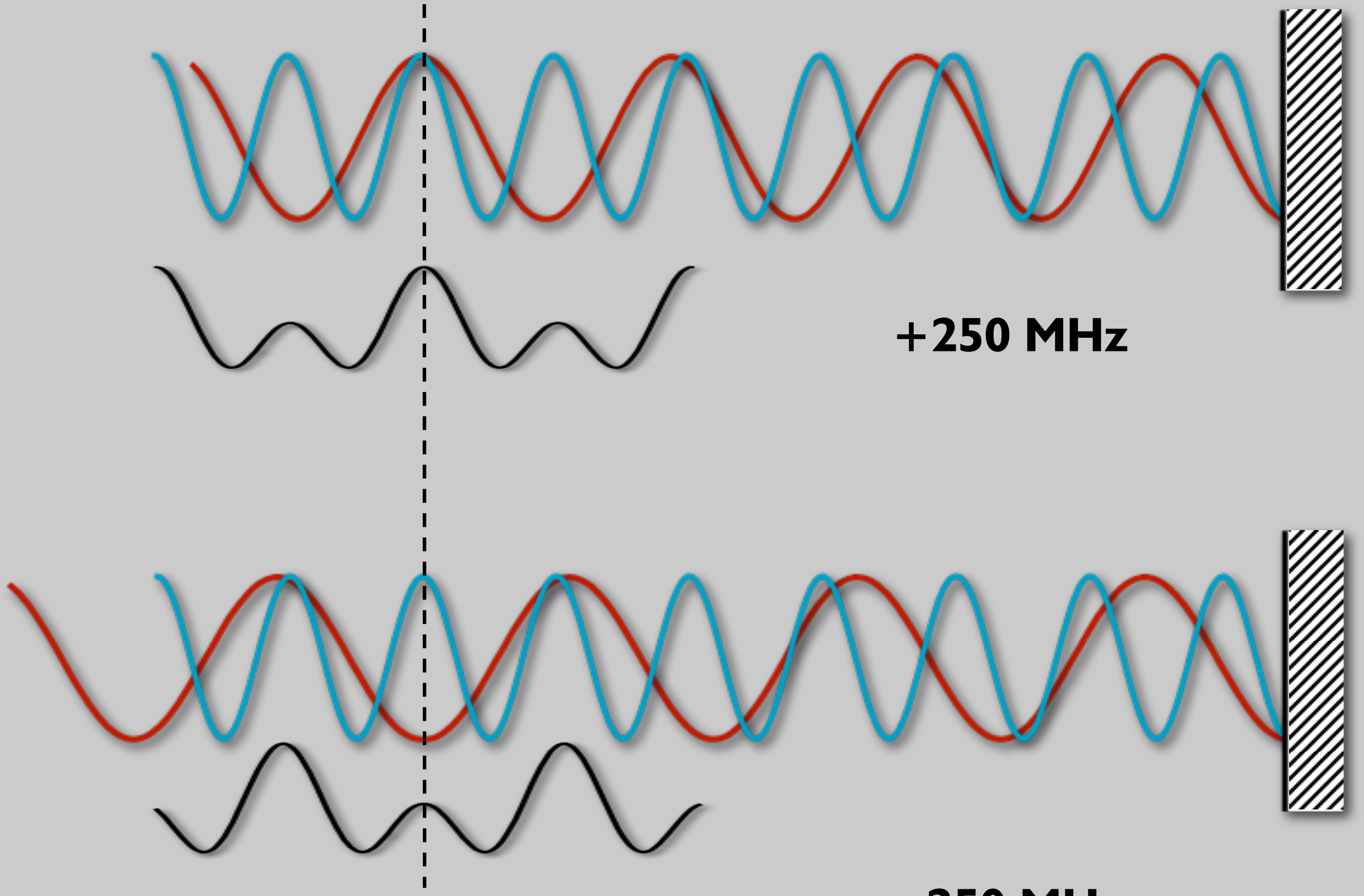
WP2,WP3,WP4,WP5,WP6

OLAQUI Review Meeting 2008,
Innsbruck

MAINZ Contributions

- **WP2 Addressing, manipulating and measuring on single sites**
  D5 Addressing single sites in optical lattices, M2.2
- **WP3 Two-qubit gates and compatible stable qubits**
  D6 Assessment of experimental feasibility for existing qubit encodings and quantum gate schemes, M3.1.1, M3.1.2, M3.1.7
  D7 Novel two-qubit gate schemes, M3.1.7
- **WP4 Generation and characterization of multi-particle entangled states**
  D8 Experimental generation of multi-particle entanglement in optical lattices, M4.1.2,M4.1.3,M4.1.7,M4.1.8
  D9 Measures and measurement procedures for multi-particle entanglement, M4.1
- **WP5 Strategies for minimizing decoherence**
  D11 Experimental realization of optical lattices with minimized decoherence M5.1,M5.2, M5.3, M5.4, M5.5, M5.6

How to make a Superlattice II

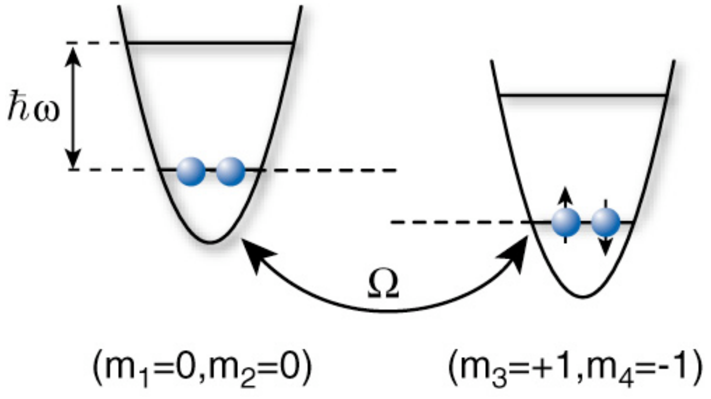


+250 MHz

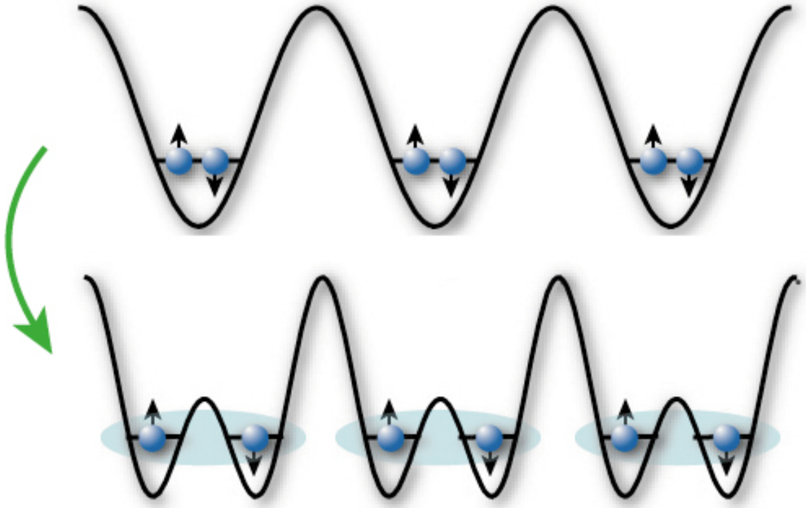
-250 MHz

see also UIBK A. Daley, G.Pupillo & P. Zoller (dressed potentials)

Robust multi-particle entanglement via spin changing collisions



A. Widera et al.,
Phys. Rev. Lett., 95,190405, (2005)



$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

Spin Triplet

$$|\uparrow\rangle_L|\downarrow\rangle_R+|\downarrow\rangle_L|\uparrow\rangle_R$$

Entangled Bell state

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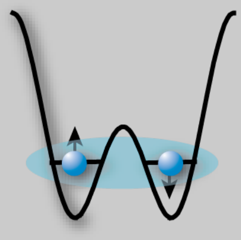
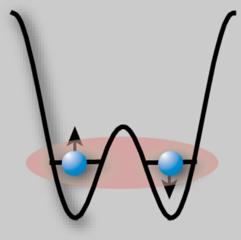
Magnetic Gradient Fields Induce Singlet-Triplet Oscillations





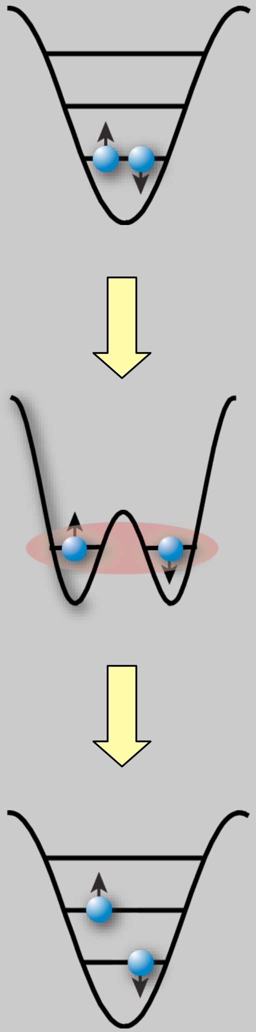
$$|\uparrow\rangle_L |\downarrow\rangle_R + |\downarrow\rangle_L |\uparrow\rangle_R \quad \Longleftrightarrow \quad |\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R$$

$$\boxed{|\uparrow\rangle_L |\downarrow\rangle_R + e^{2i\phi(t)} |\downarrow\rangle_L |\uparrow\rangle_R}$$

WP3 D6,7

$$\phi(t) = \mu_B B' d_{DW}$$

How can we detect the Bell pairs? (2)



$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

Split

$$|\uparrow\rangle_L|\downarrow\rangle_R+|\downarrow\rangle_L|\uparrow\rangle_R$$

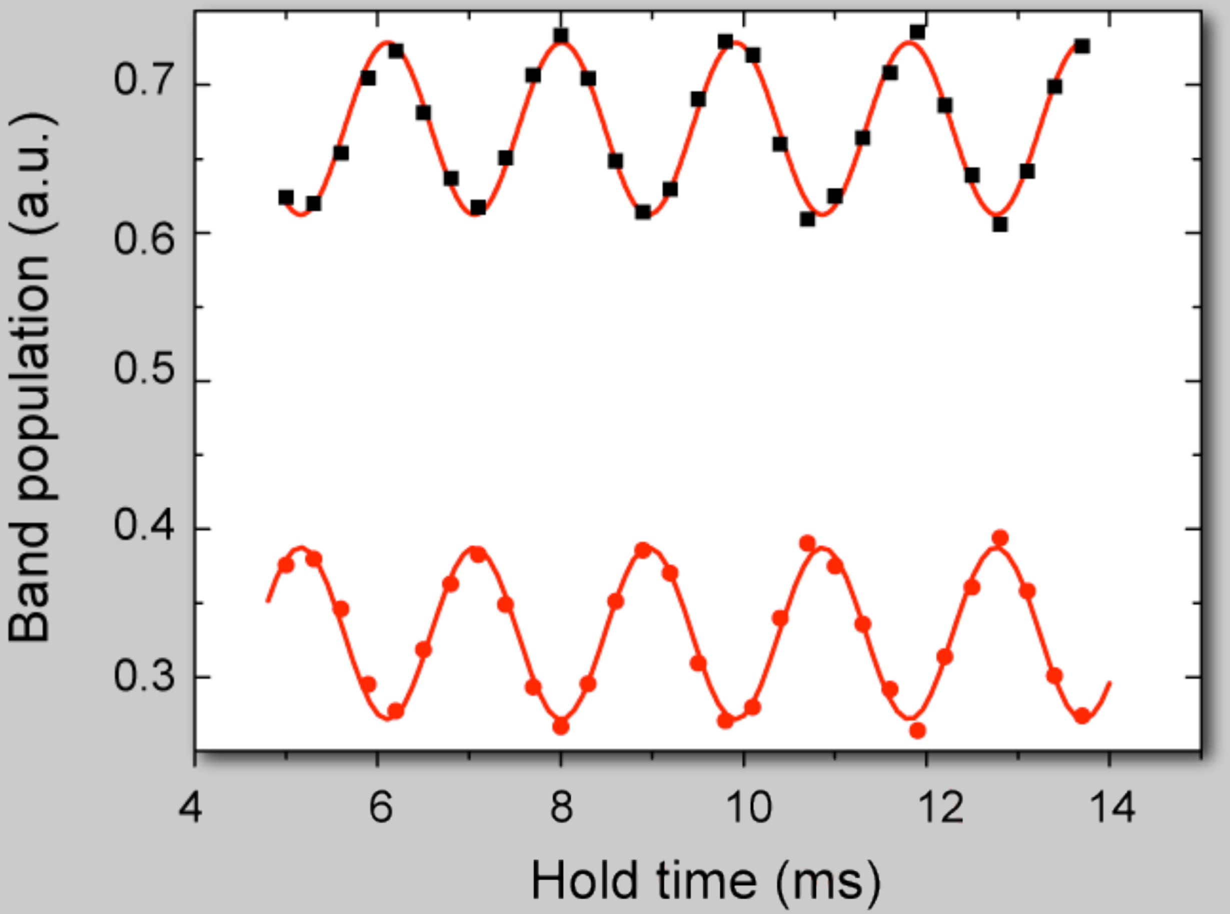
Unite

$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes\left(|0,1\rangle+|1,0\rangle\right)$$

When uniting bosonic spin singlet states, one particle has to occupy the excited band!

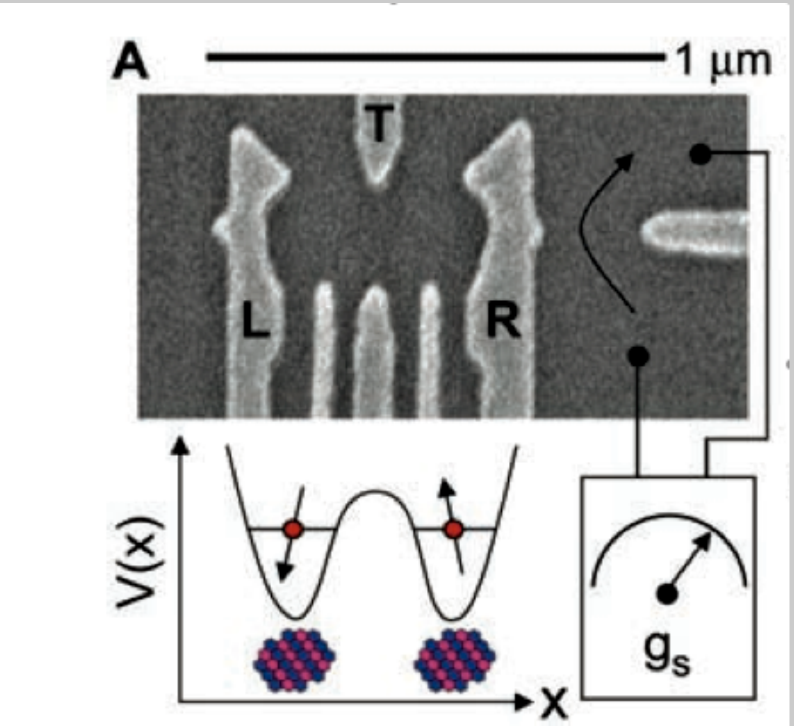
Singlet-Triplet Spin Oscillations

S. Trotzky (in preparation)



See ion trap exps: C.F. Roos et al., PRL **92**, 220402 (2004),
C. Langer et al., PRL **95**, 060502 (2005)

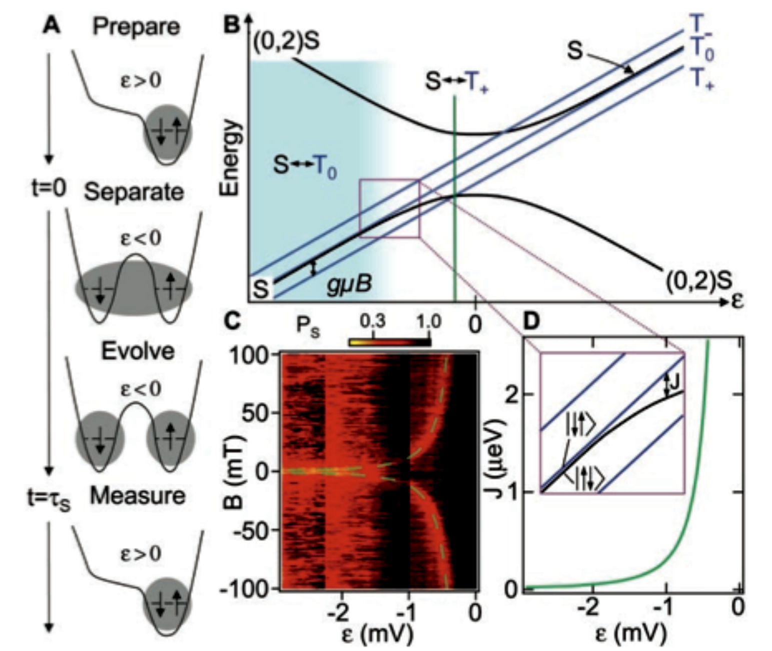
Superexchange Coupling in Quantum Dots



Local control of spin states & interactions between spin states.

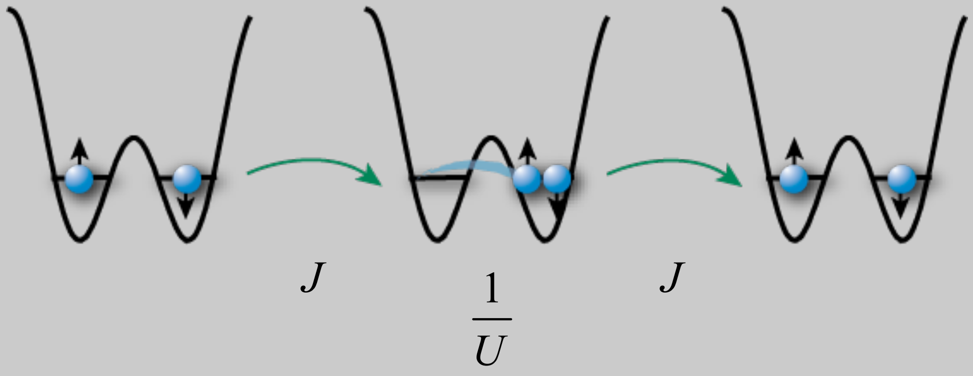
J.R. Petta et al., Science **309**, 2180 (2005)

Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots



Quantum Magnetism

Second order hopping processes form the basis of superexchange interactions! (see e.g. A. Auerbach, Interacting Electrons and Quantum Magnetism)



$$-J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j+\frac{1}{2}U\sum_i\hat{n}_i(\hat{n}_i-1)$$

$$\boxed{H=-J_{ex}\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j}$$

$$\boxed{J_{ex}\propto\frac{J^2}{U}}$$

Ultracold atoms allow tuning of Spin-Hamiltonians

$$H=\sum_{\langle i,j\rangle}\left[\lambda_{\mu z}\hat{\sigma}_i^z\hat{\sigma}_j^z\pm\lambda_{\mu\perp}\left(\hat{\sigma}_i^x\hat{\sigma}_j^x+\hat{\sigma}_i^y\hat{\sigma}_j^y\right)\right]$$

$$\lambda_{\mu z}=\frac{t_{\mu\uparrow}^2+t_{\mu\downarrow}^2}{2U_{\uparrow\downarrow}}-\frac{t_{\mu\uparrow}^2}{U_\uparrow}-\frac{t_{\mu\downarrow}^2}{U_\downarrow}$$

$$\lambda_{\mu\perp}=\frac{t_{\mu\uparrow}t_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

L.M. Duan et al., PRL **91**, 090402 (2003),
E. Altman et al., NJP **5**, 113 (2003), A.B. Kuklov et al. PRL **90**, 100401 (2003)

Superexchange induced flopping

$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$

$$= -\frac{J_{ex}}{2}\left(\hat{S}_i^+\hat{S}_j^- + \hat{S}_i^-\hat{S}_j^+\right) - J_{ex}\hat{S}_i^z\hat{S}_j^z$$

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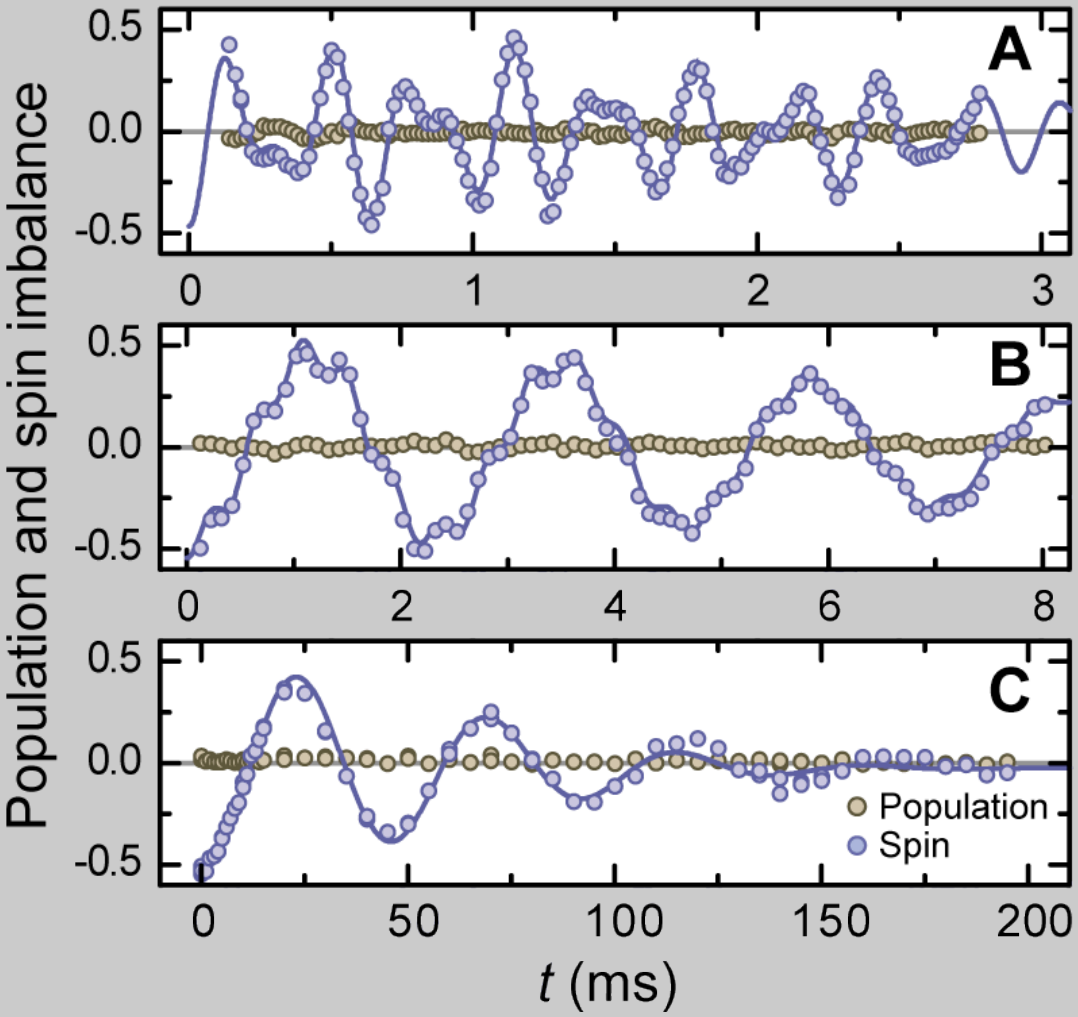
Superexchange induced flopping



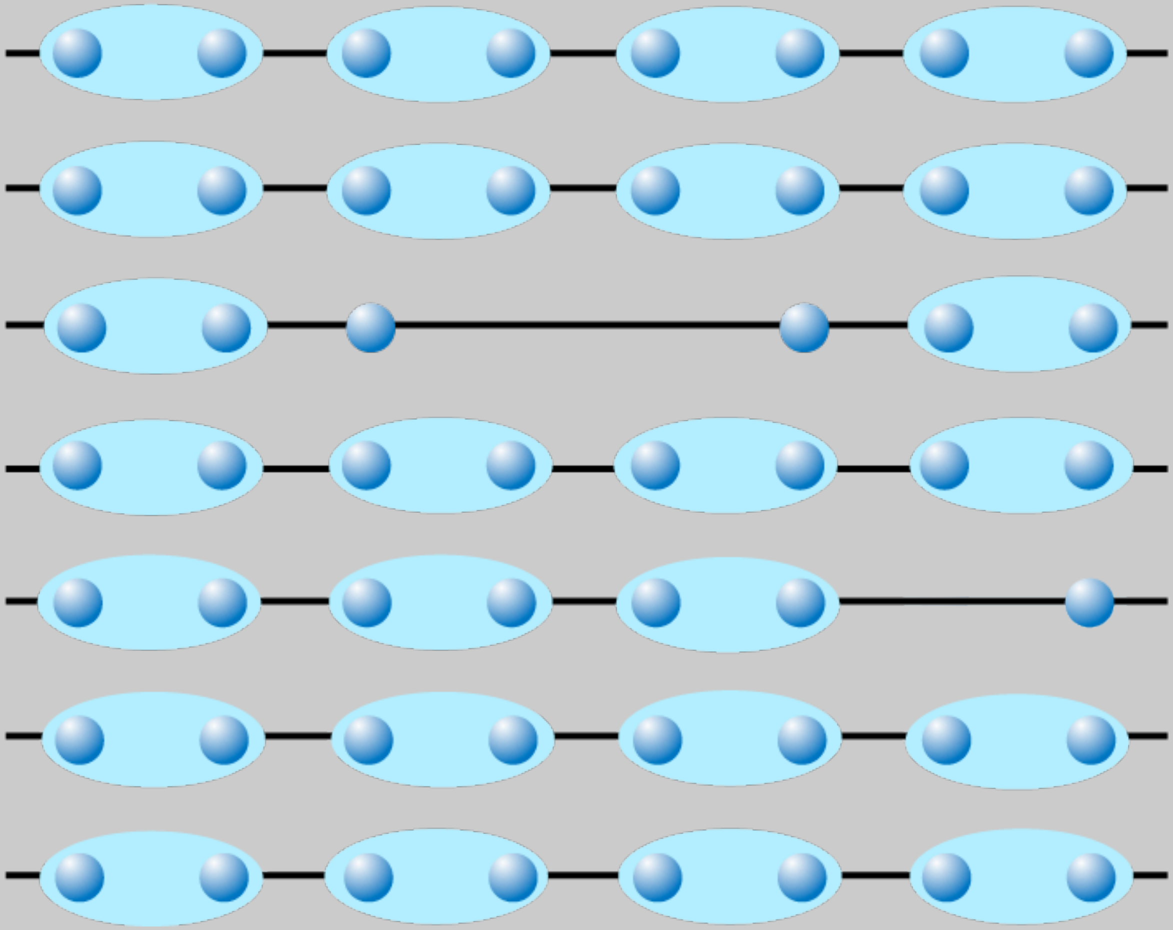
$J/U = 1.25$
$V_{short} = 6\ E_r$

$J/U = 0.26$
$V_{short} = 11\ E_r$

$J/U = 0.05$
$V_{short} = 17\ E_r$

S. Trotzky et al. Science **319**, 295 (2008)

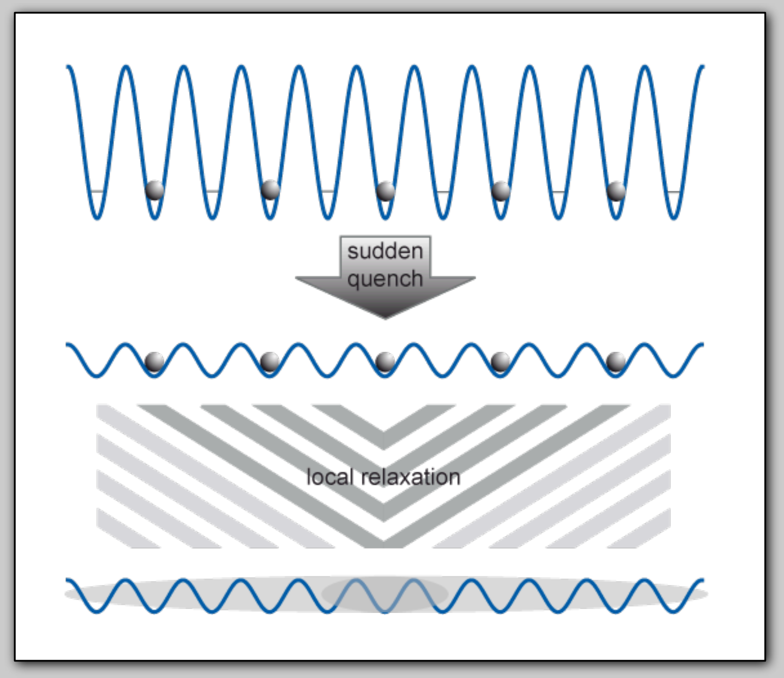
Arrays of Entangled Bell Pairs



A. M. Rey et al., PRL 99, 140601 (2007),
B Vaucher, A Nunnenkamp and D. Jaksch, NJP 10, 023005 (2008)

Relaxation Dynamics

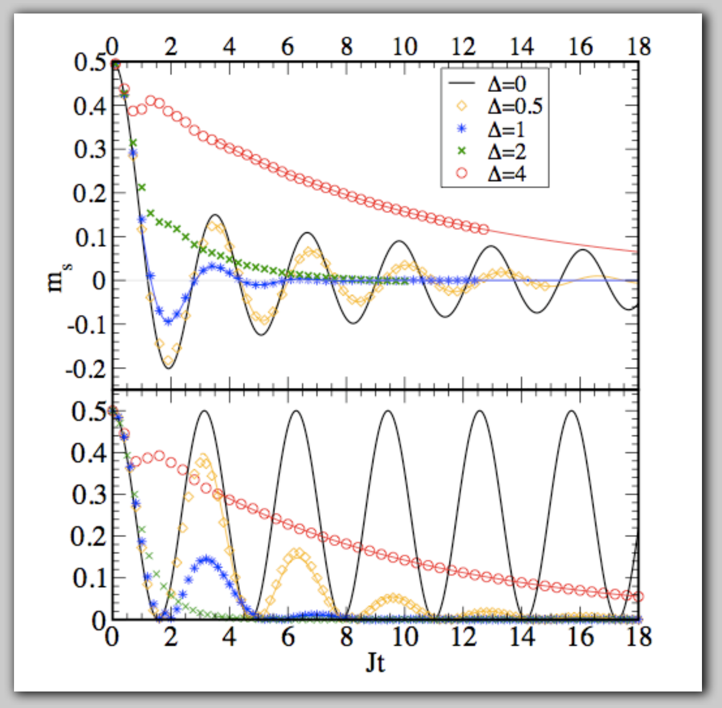
Relaxation of CDW - how is equilibrium approached?



J. Eisert, U. Schollwöck

Full Hubbard dynamics difficult
to solve with DMRG

Relaxation of Néel state under
XXZ model

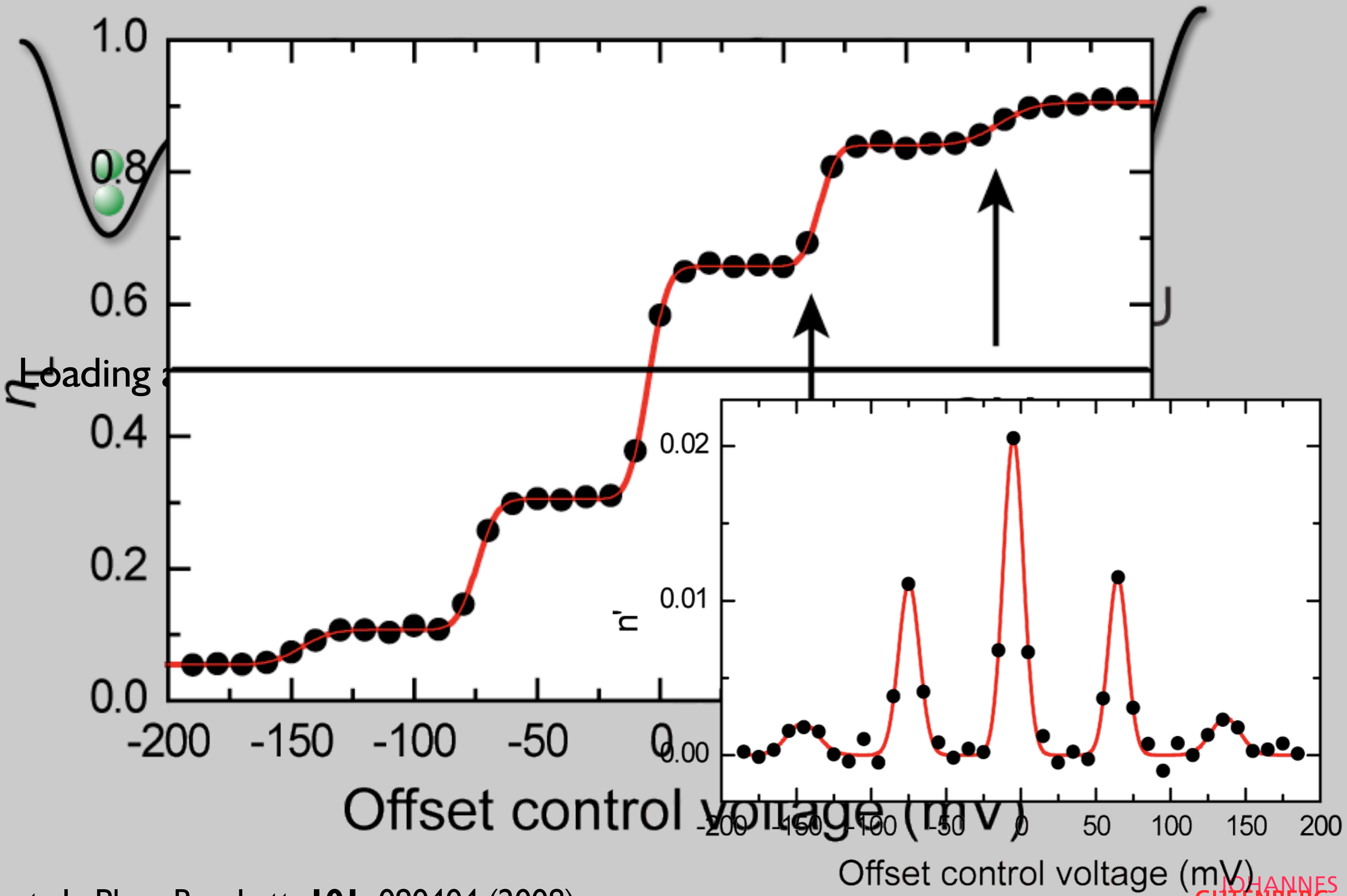


$$H_{XXZ} = J_{ex} \sum_j \left\{ S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right\}$$

P. Barmettler, E. Demler, E. Altman & M. Hastings, L. Levitov

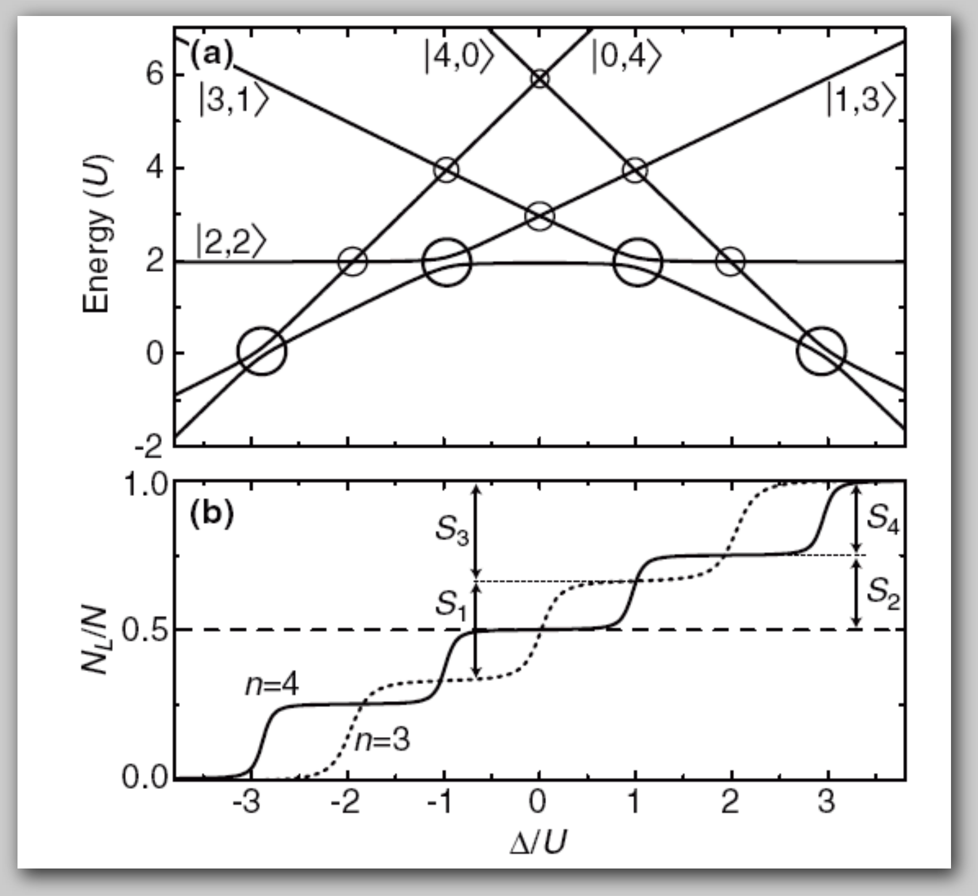
Counting Atoms
using
Interaction Blockade Induced Tunnelling Resonances

„Coulomb“ Blockade Type Tunnelling Resonances



P. Cheinet et al., Phys. Rev. Lett. **101**, 090404 (2008)

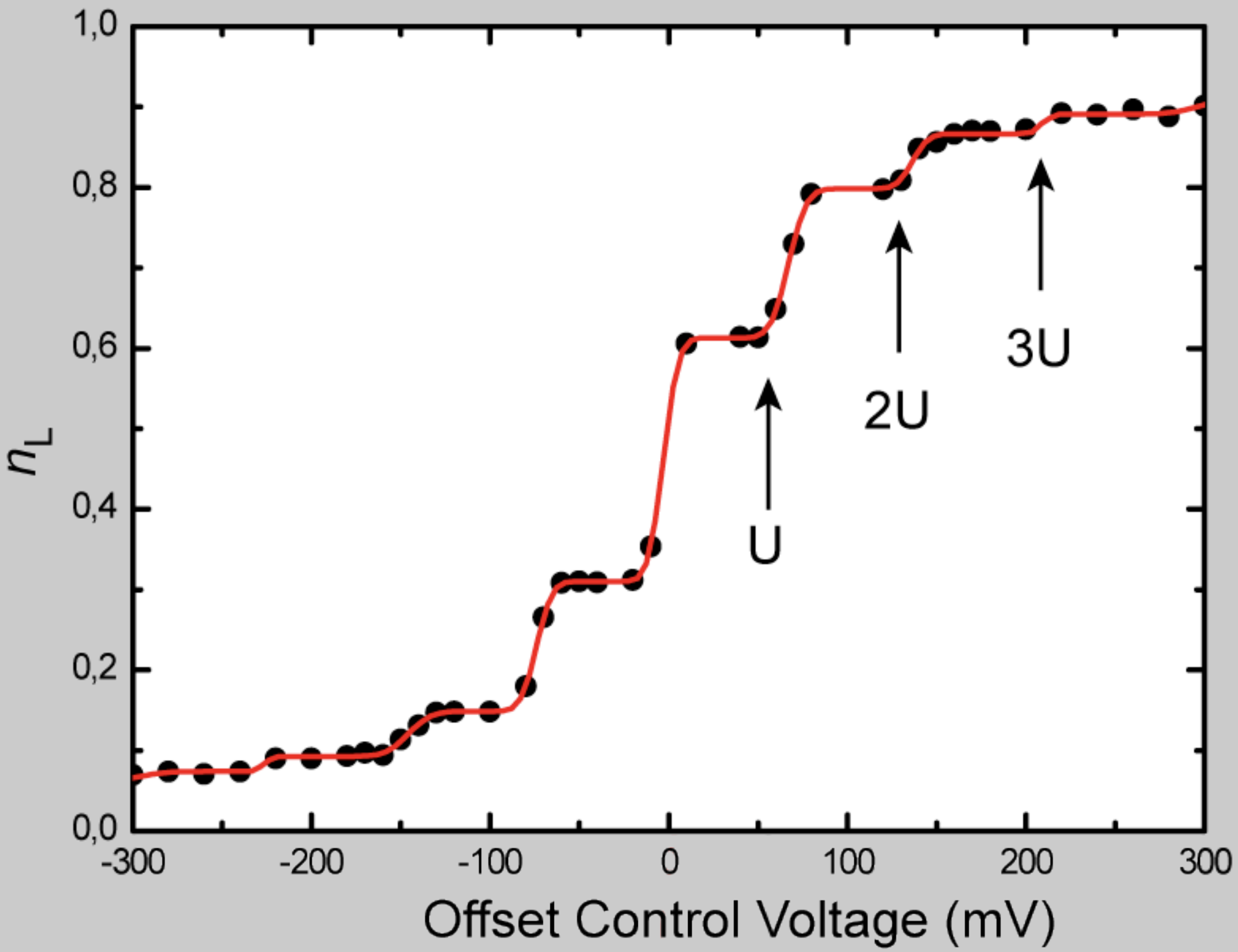
Energy Diagram



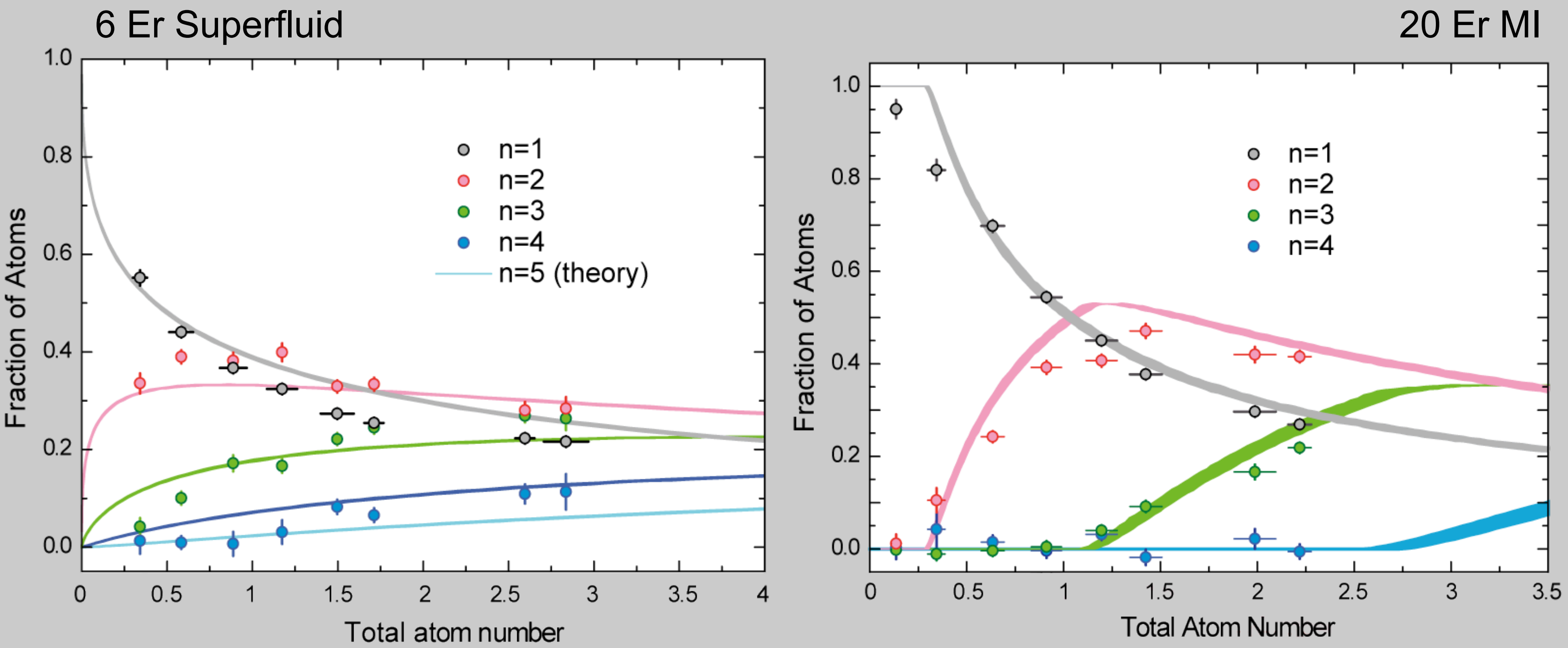
Entanglement generation possible for non-adiabatic crossings!

Higher order tunnelling: S. Fölling et al., Nature **448**, 1029 (2008)
P. Cheinet et al., Phys. Rev. Lett. **101**, 090404 (2008)

higher orders....



Measuring Atom Number Statistics


6 Er Superfluid

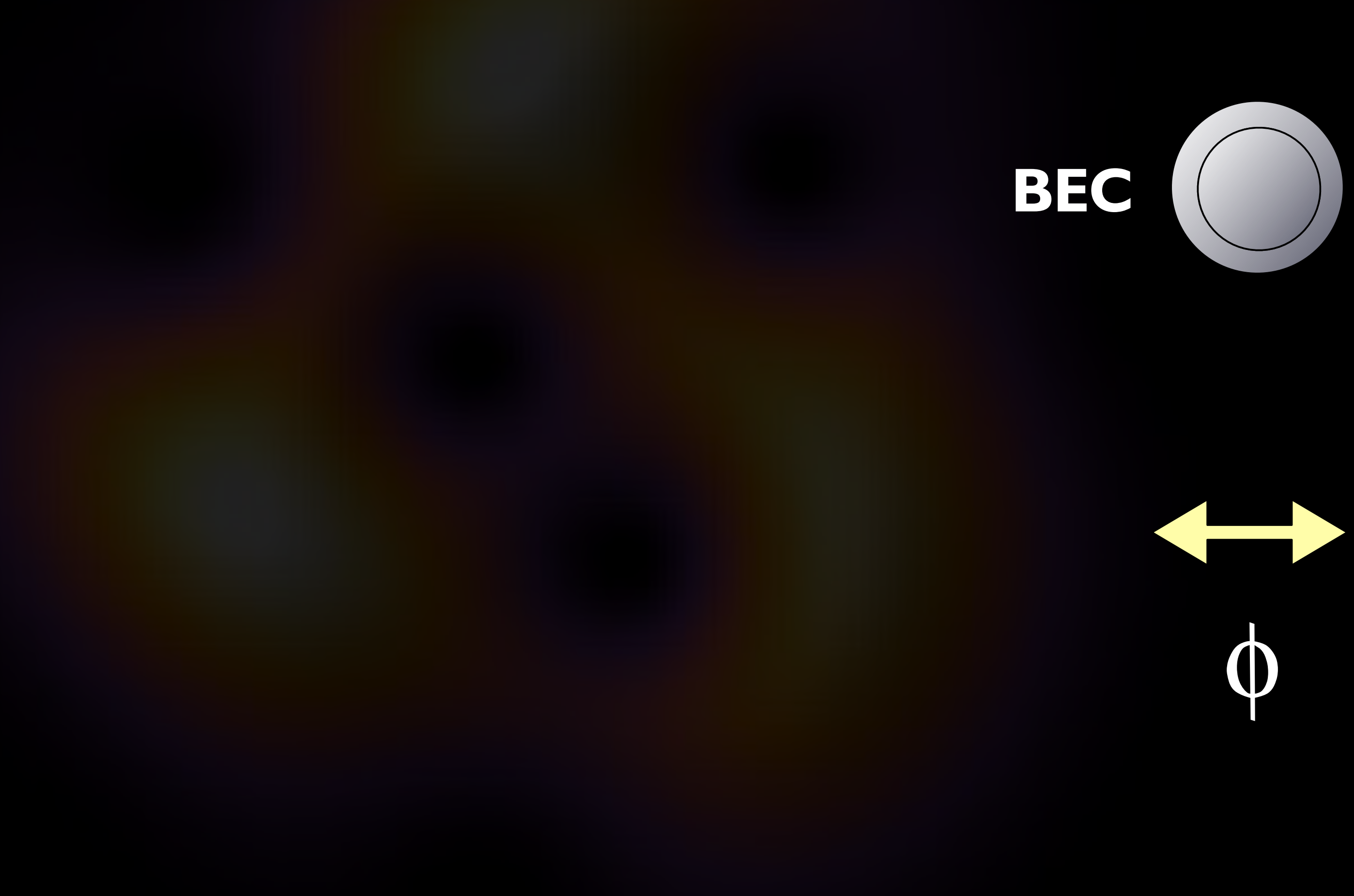
20 Er MI

Phase diffusion: Greiner et al., Nature 2002, Sebby-Strabley et al., PRL 2007
Spin changing collisions: Fölling et al., PRL 2006
Spectroscopy: Campbell et al. Science 2006, Jördens et al., Nature 455, 204 (2008)

Quantum Phase Diffusion and Spin Squeezing

Contribution to WP4

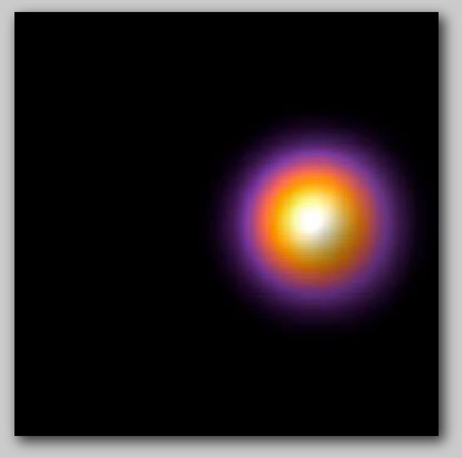
Probing Many-Body States via Quantum Phase Diffusion



Phase Diffusion Dynamics

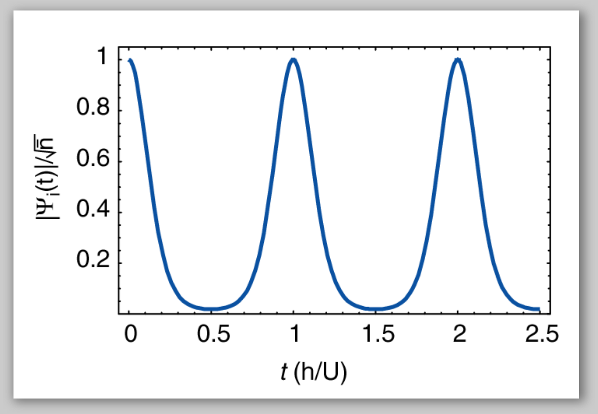
Quantum state in each lattice site (e.g. for a coherent state)

$$\left|\Psi(t)\right\rangle_i = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}\, e^{-i\frac{1}{2}U\,n(n-1)\,t/\hbar}\left|n\right\rangle$$



Matter wave field on the i^{th} lattice site

$$\Psi_i(t) = {}_i\left\langle\Psi(t)\right|\hat{a}_i\left|\Psi(t)\right\rangle_i$$



1. ***Matter wave field collapses but revives after times multiple times of h/U !***
2. ***Collapse time depends on the variance σ_N of the atom number distribution !***

Theory: Yurke & Stoler, 1986, F. Sols 1994; Wright et al. 1997; Imamoglu, Lewenstein & You et al. 1997, Castin & Dalibard 1997, E. Altman & A. Auerbach 2002,

Exp: M. Greiner et al 2002, G.-B. Jo et al 2006, J. Sebby-Strabley et al. 2007, M. Oberthaler

Similiar to Collapse and Revival of Rabi-Oscillations in Cavity QED !

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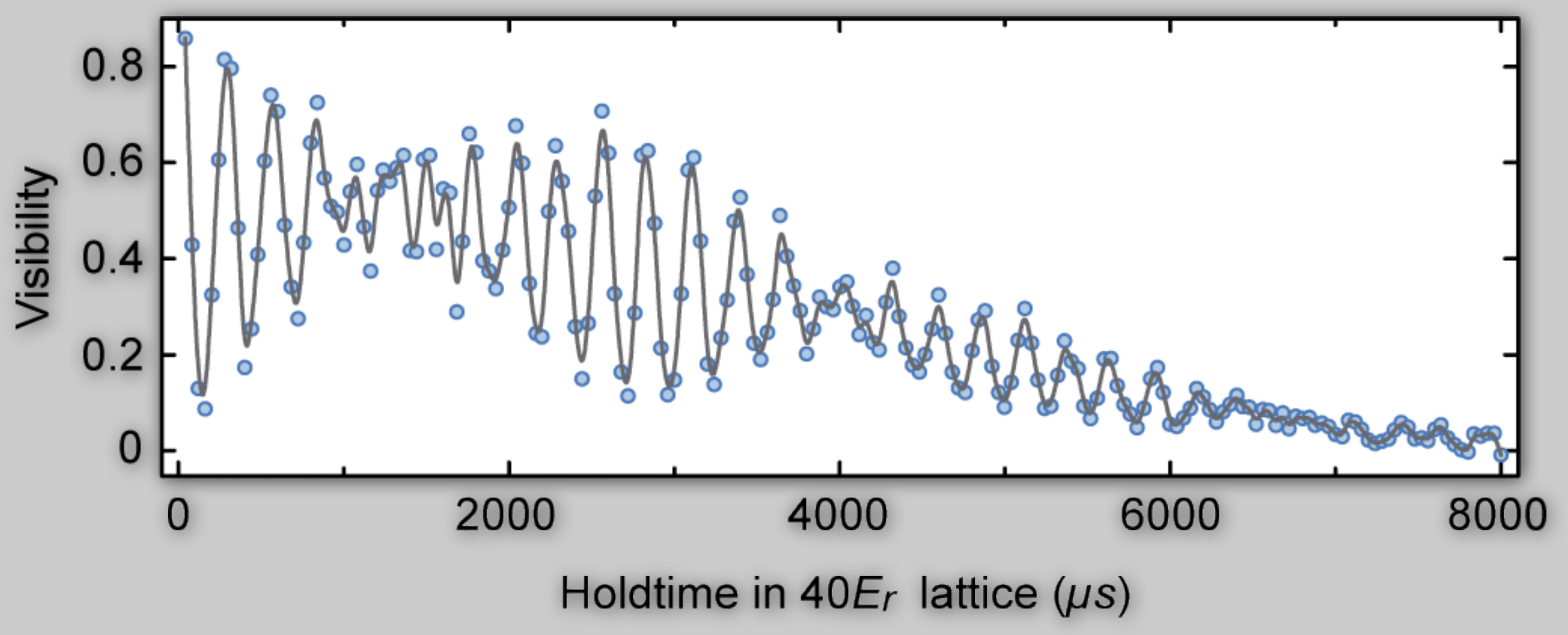
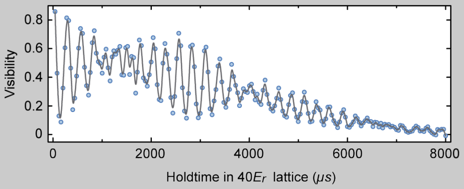
Collapse & Revivals of a BEC under Homogeneous External Confinement



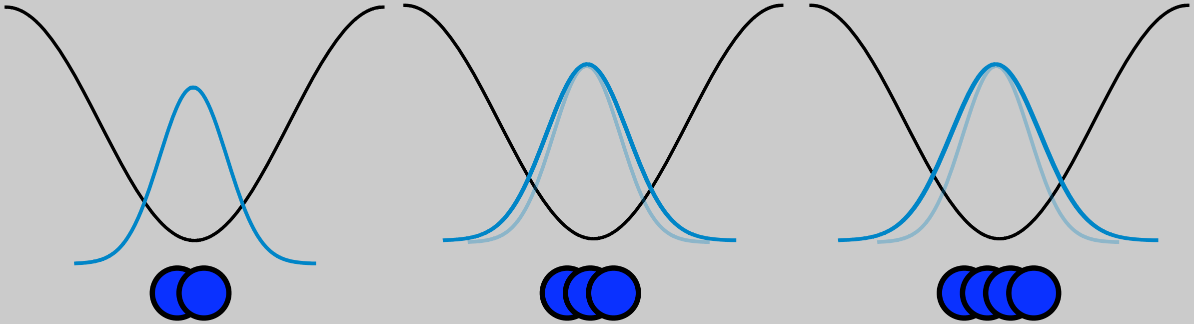
- Up to 40 Revivals Detectable!
- Multiple Frequency Components!



Why Multiple Frequencies?

$$\left|\Psi(t)\right\rangle_i = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}\, e^{-i\frac{1}{2}U\,n(n-1)t/\hbar}\left|n\right\rangle$$

We assume U to be constant, independent of filling....



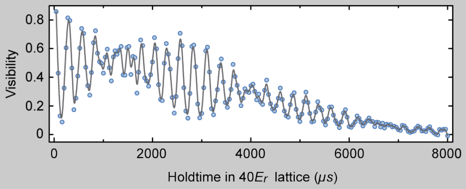
n=2 n=3 n=4

U(2) U(3) U(4)

$$U = \frac{4\pi\hbar^2 a}{m}\int d^3x\,|w(\mathbf{x})|^4$$

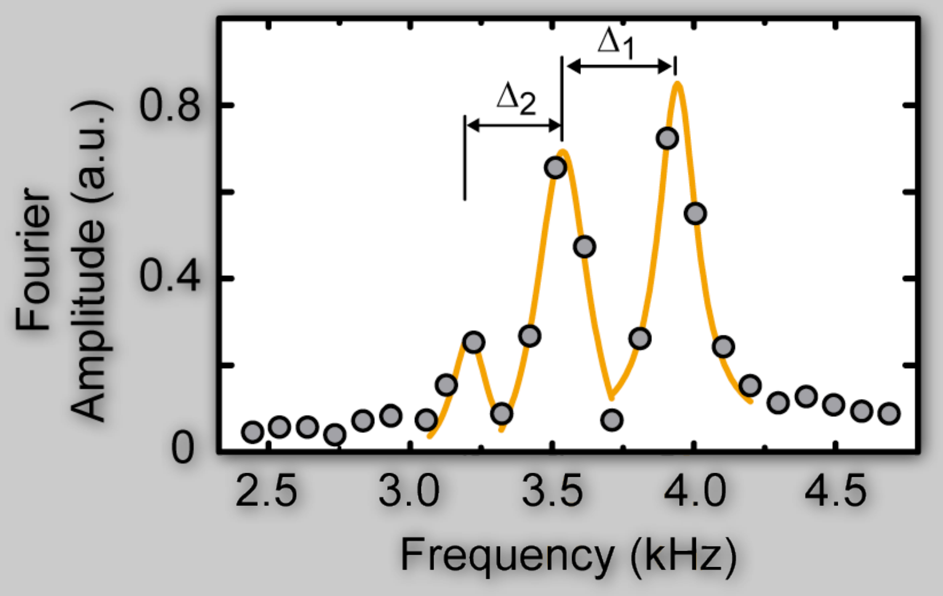
Breakdown of single band approximation!

for differential measurement, see also: G. Campbell et al. Science (2006)



Fourier Spectrum

$$\langle\alpha|\hat{a}|\alpha\rangle = \alpha\left\{p_0 + p_1 e^{-iU(2)t/\hbar} + p_2 e^{-i(3U(3)-U(2))t/\hbar} + p_3 e^{-i(6U(4)-3U(3))t/\hbar} + \ldots\right\}$$



$$\hbar\Delta_1 = 3\Delta U(3)$$

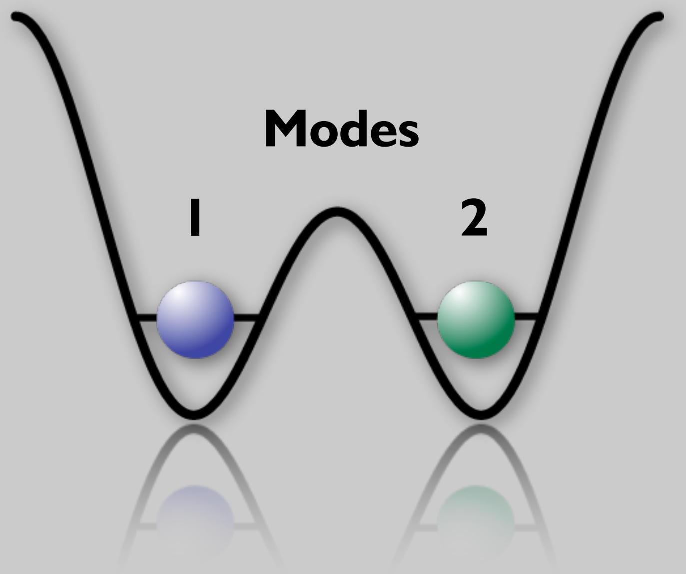
$$\hbar\Delta_2 = 6\Delta U(4)$$

$$U(3) = U(2) + \Delta U(3)$$

$$U(4) = U(3) + \Delta U(4)$$

Phase Diffusion in Spin Language

$$H = -J\hat{S}_x + U\hat{S}_z^2$$

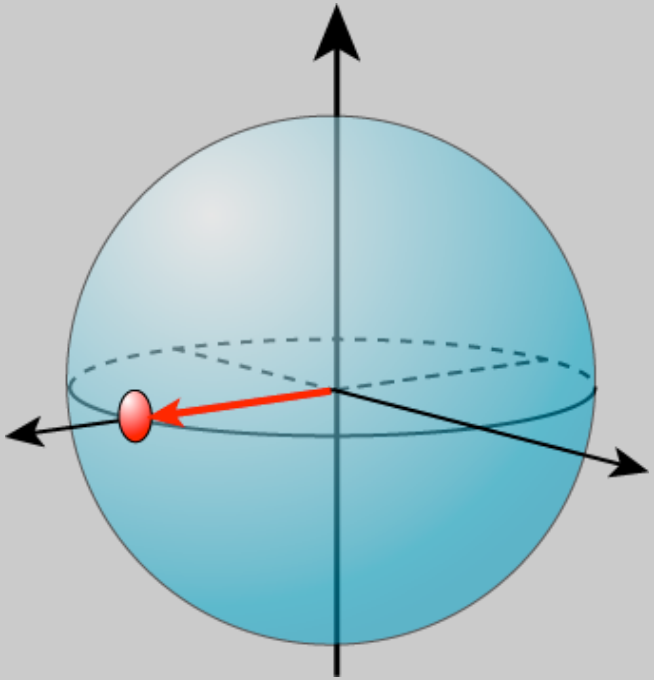


$$\hat{S}_x = (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)/2$$

$$\hat{S}_y = (\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)/2$$

$$\hat{S}_z = (\hat{n}_1 - \hat{n}_2)/2$$

From Spin Squeezing to Schrödinger Cats
- Nonlinear Quantum Spin Dynamics -



What happens if you tune interactions in larger ensembles?

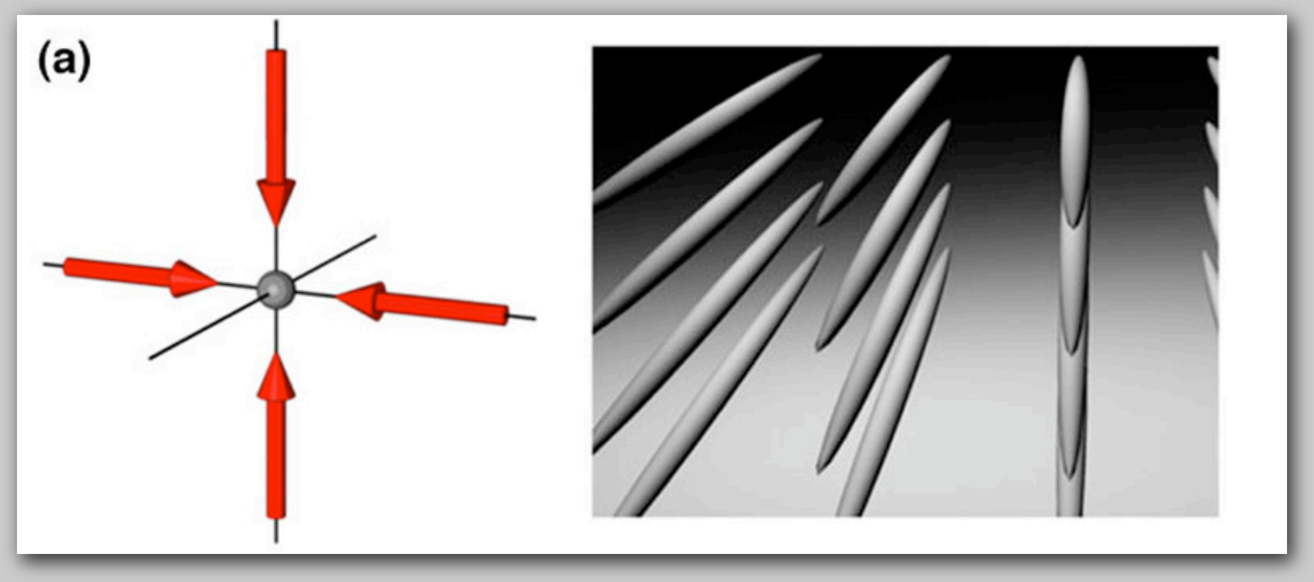
$$\left(\hat{a}^\dagger + \hat{b}^\dagger\right)^{\otimes N} |0\rangle$$

$$\hat{H} = \chi \hat{S}_z^2$$

$$\chi = a_{aa} + a_{bb} - 2a_{ab}$$

A. Sørensen et al., Nature 409, 63 (2001),
L. You, PRA (2002)
A. Micheli et al. PRA 67, 013607 (2003)
see also work of M. Oberthaler, J. Schmiedmayer,
D.Pritchard & W. Ketterle

Nonlinear Spin Dynamics in 1D Quantum Spin Systems

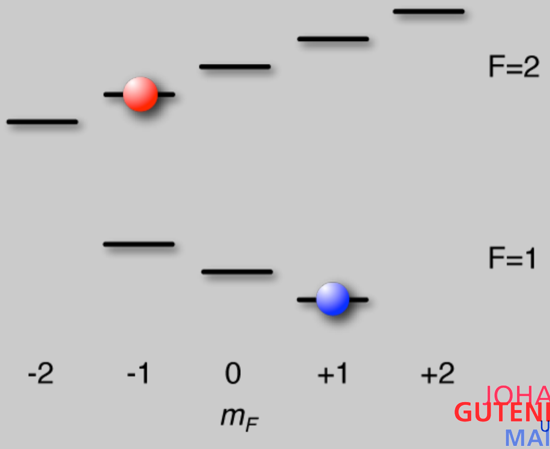


System Parameters:

Atom Number: N=60 per tube

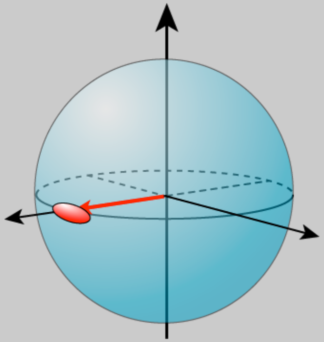
Trapping Frequencies: $\nu_r = 42\,\mathrm{kHz}$ $\nu_{ax} = 90\,\mathrm{Hz}$

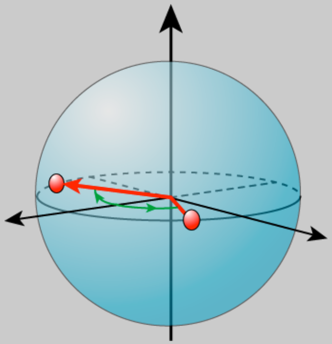
Luttinger Parameter: $\gamma \simeq 0.1 - 0.2$ $K \simeq 5 - 8$



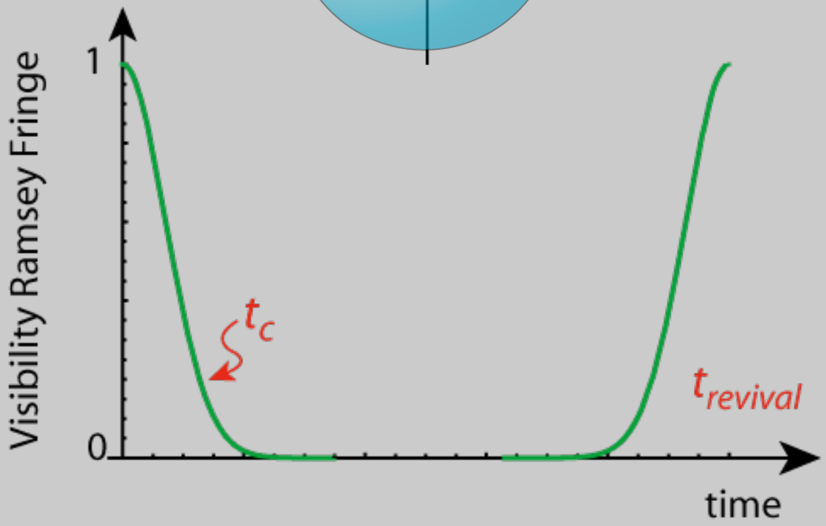
Ramsey Fringe Visibility Evolution

π/2 → **Tune interactions and wait** → π/2





$$\frac{N_{\uparrow}}{N_{tot}} = \frac{1}{2}\left(1 + \mathscr{V}(t)\cos(\theta)\right)$$



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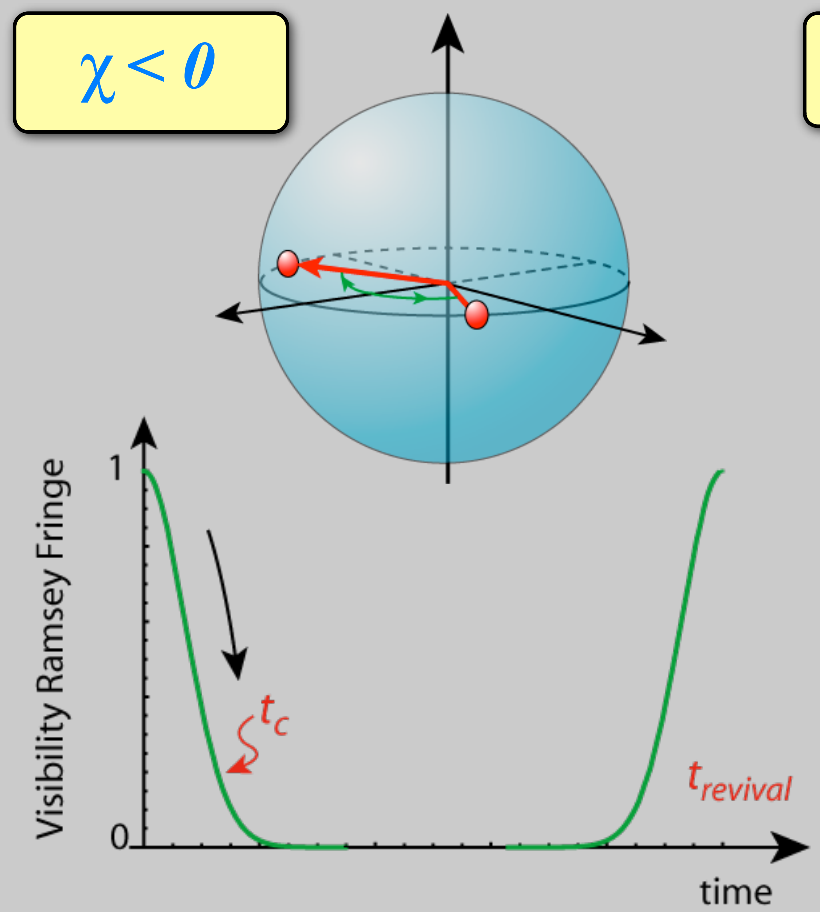
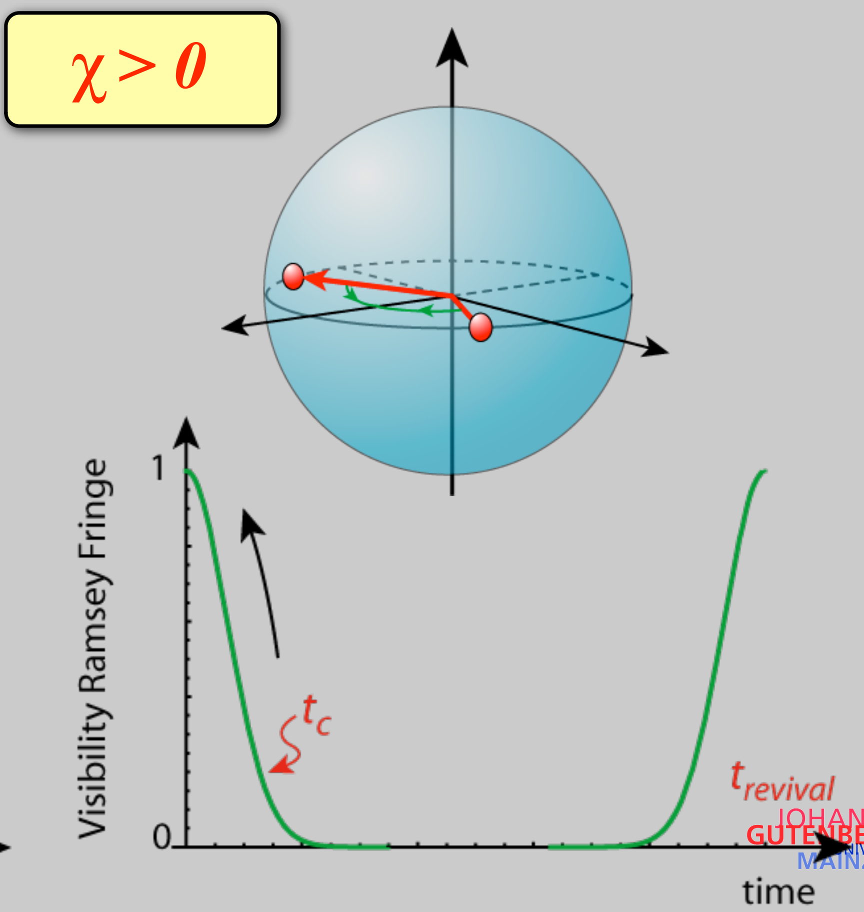
Decoherence Problems

Full revival can so far not be reached due to decoherence, but time evolution can be reversed!

$\chi < 0$



$\chi > 0$



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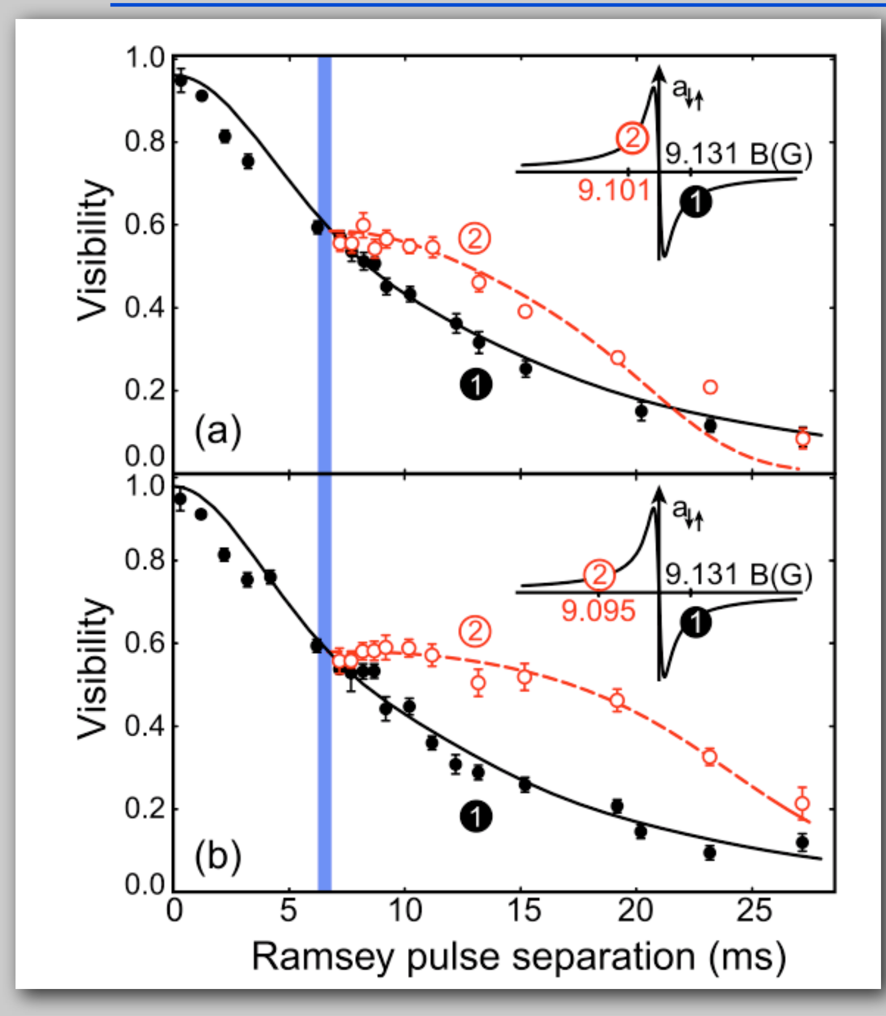
Reversing the Coherent Dynamics



Dynamics can only be partly reversed…

Why?

A. Widera et al. Phys. Rev. Lett. (2008)

Quantum Fluctuations in 1D!

Predict dynamical evolution using a Luttinger Liquid approach (effective Hamiltonian for low energy behaviour of 1D system) (cp. Haldane)

Here we need to employ a two component version of Haldane's approach!

For $a_{\uparrow\uparrow} \approx a_{\downarrow\downarrow}$ ***elementary excitations decouple into independent density and spin part!***

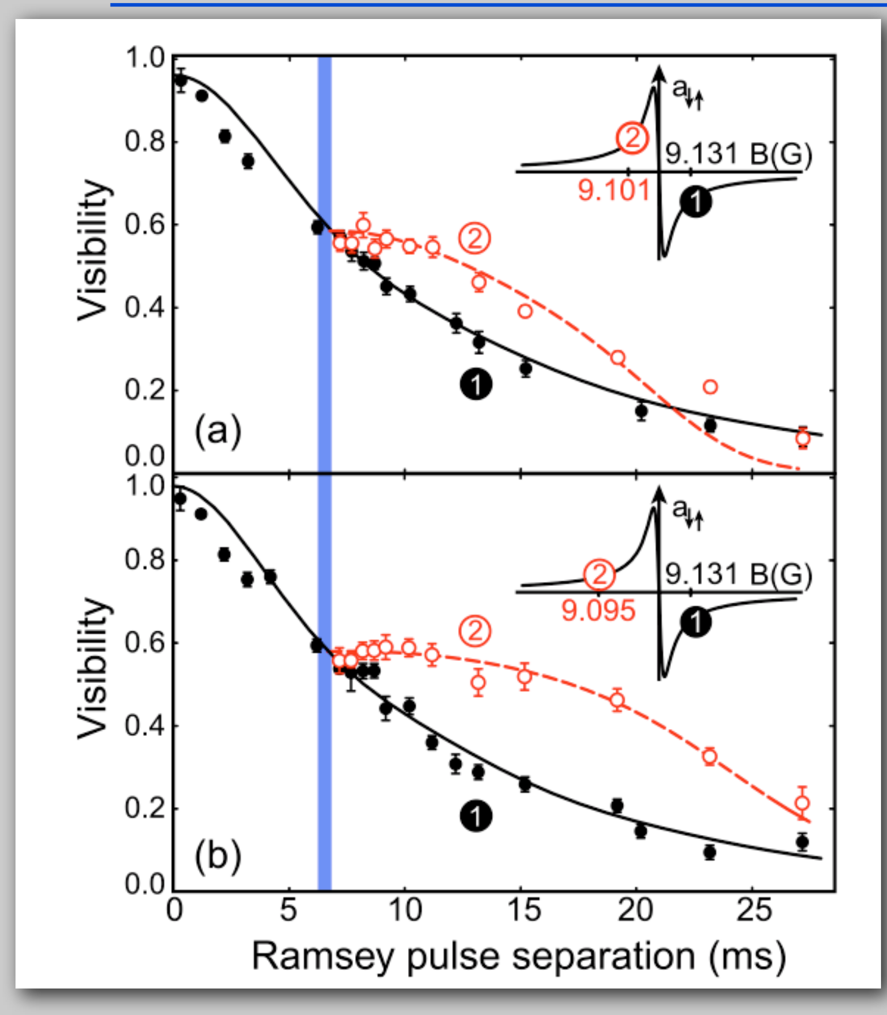
Spin part:

$$\boxed{H_S = \int dx \left[g_s \hat{m}_z^2 + \frac{n_{tot}}{4M} \left(\nabla \hat{\phi}_s \right)^2 \right]}$$

$$\begin{aligned} \hat{\phi}_s(x) &= \phi_0 + \Sigma_{q\neq 0} (2qLK/\pi)^{-1/2} \\ &\quad e^{-|q|/2q_c}(q) \left[e^{iqx} \hat{a}_q + h.c. \right] \end{aligned}$$

By *reversing the interaction energy*, we *only* completely reverse the dynamics for the *q=0 mode*!

Reversing the Coherent Dynamics



A. Widera et al. PRL (2008)
see also R. Bistritzer & E. Altman PNAS 2007, and
work of J. Schmiedmayer, Th. Schumm (Vienna)

Dynamics can only be partly reversed...

interaction reversal does not take into account quantum fluctuations in 1D

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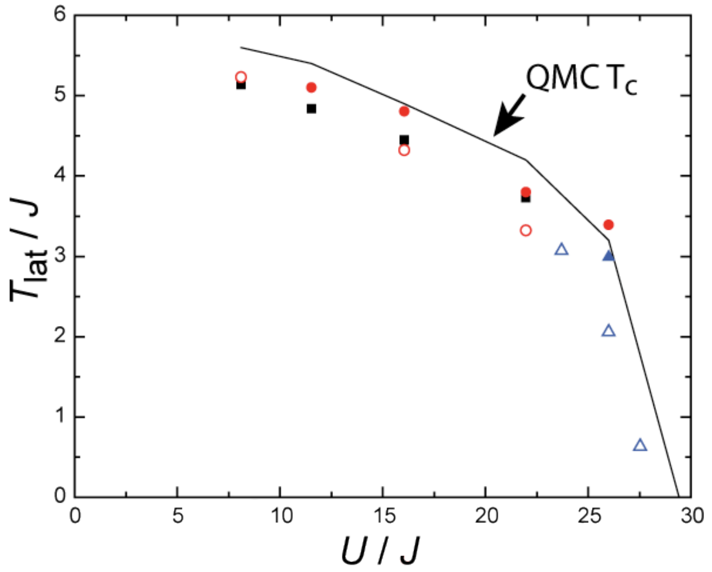
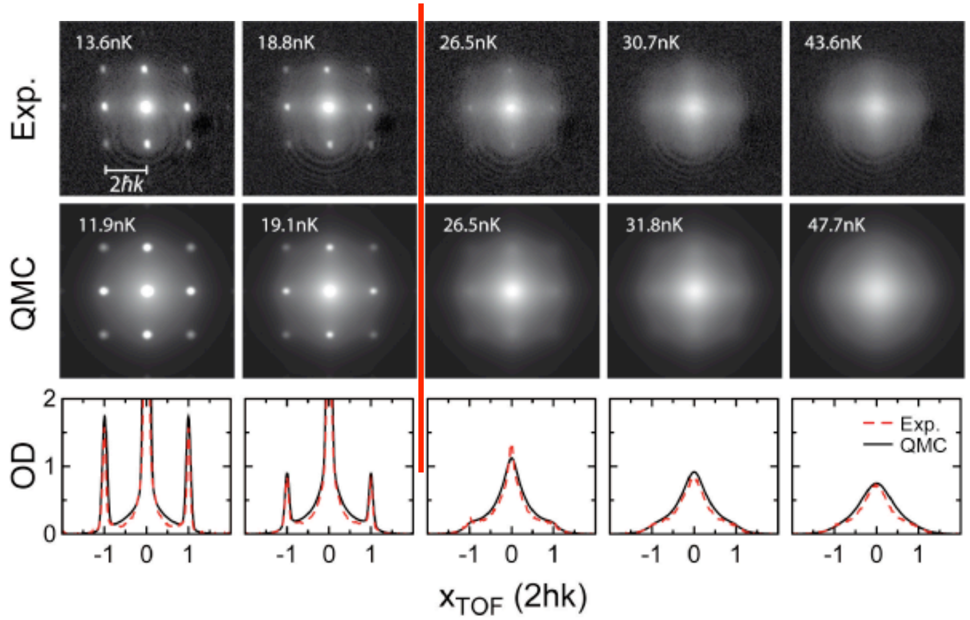
Quantum Simulations
WP6

Superfluid-Mott Insulator Transition
QMC vs Exp.



Vanishing of Tc when QCP is approached

Momentum distributions for U/J=8.11



ETHZ, UMass, Mainz
joint theory-exp project

S. Trotzky, L. Pollet et al. (in preparation)

MAINZ Contributions

- **WP2 Addressing, manipulating and measuring on single sites**
  D5 Addressing single sites in optical lattices, M2.2
- **WP3 Two-qubit gates and compatible stable qubits**
  D6 Assessment of experimental feasibility for existing qubit encodings and quantum gate schemes, M3.1.1, M3.1.2, M3.1.7
  D7 Novel two-qubit gate schemes, M3.1.7
- **WP4 Generation and characterization of multi-particle entangled states**
  D8 Experimental generation of multi-particle entanglement in optical lattices, M4.1.2,M4.1.3,M4.1.7,M4.1.8
  D9 Measures and measurement procedures for multi-particle entanglement, M4.1
- **WP5 Strategies for minimizing decoherence**
  D11 Experimental realization of optical lattices with minimized decoherence M5.1,M5.2, M5.3, M5.4, M5.5, M5.6

Summary & Outlook

- Creation and loading of atoms in optical superlattices
- Single Qubit State Manipulation
- Massively Parallel Creation of Bell Pairs
- Characterization of Bell Pairs
- Measurement of Coherence Time of Single Qubits and Bell Pairs
- Controllable Superexchange Spin-Spin Interaction
- Novel Multiparticle Entanglement Schemes for Generation of Robust MP Entangled Quantum States
- Multiparticle Entanglement via Spin-Squeezing in 1D Quantum Systems
- Investigation of Dynamical Effects in Mode-Squeezed 1D Luttinger Liquids
- Interaction Blockade Mechanism to Count Single Atoms/Number Statistics
- Collapse & Revival for cat state generation/number state resolution achieved
- Quantum Simulations (Fermionic Atoms/Bosonic Atoms)
- Light Storage/Generation in Mott Insulators